From pebbles to digital signs -

the joint origin of signs for numbers and for scripture, their intercultural standardization and their renewed conjunction in the digital era

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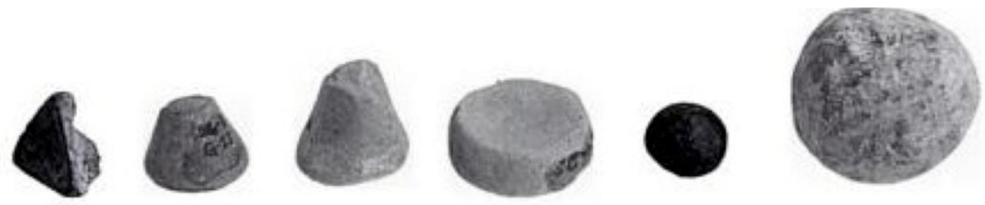
Introduction

The historical development until the present-day almost universal encoding of information has been extraordinarily complex. Encodings began as concrete materializations and were intimately tied to entirely specific social and cultural forms of living. Characteristic stages of encodings, from highly differentiated material sign systems towards abstract and globally used symbolizations will be presented and analyzed. A particularly revealing approach to characteristic patterns of these transformations is constituted by the dimension how the two encoding systems – numbers and scripture which used to develop over millennia in separated ways were related to each other: hence how numeracy and literacy were and are related.
It has to be remarked that the implied processes of universalization and standardization were by no means occurring in a smooth or easy way; rather they used to be confronted with considerable resistance, due to the weight of local and regional traditions.

Mesopotamia

The emergence of sign systems for scripture and for numbers has best been studied for the cultures in Mesopotamia, thanks to the durable material used there for writing and calculating. The first artefacts used there were quite different from any known form of writing and archaeologists therefore did not pay much attention to them for a long time. They were seemingly strange material objects, not yet produced by clay but from stone, cut into appropriate but not at all standardized forms: the Greeks would use them again for certain means (see below) and call them ψηφοι (psephoi). The Romans used them in a standardized form for calculating and called them *calculi* – the origin of the calculus resides hence in the oldest known artefacts for sign systems! The English term for them is pebbles or tokens.[1]

[1] The oral talk will present a great number of illustrations. For this written version, only a small selection can be presented.



The tokens indicated at the same time a quantity and a quality: they signified an object (quality) and its size (quantity). They were laid on the top of each container, the form as a sign for the objects in the container and the size for the amount. From the earliest times of Mesopotamian states, from more than 3.000 BC, the artefacts were used for administrating the goods delivered by the tributaries to the temples, the then centers of state administration and hence also of economy.

With the evolution of the administrational experience, the tokens became more specific for indicating qualities and quantities, in particular since the emergence of the technique to use clay tablets for engraving signs, so that bookkeeping could occur detached from the place of storage. Modern research succeeded in establishing the list of signs used in Uruk, an early high civilization state of the third millennium BC in Mesopotamia, used there for this bookkeeping. This is the list of sixty signs, evidencing sophisticated cuneiform techniques:

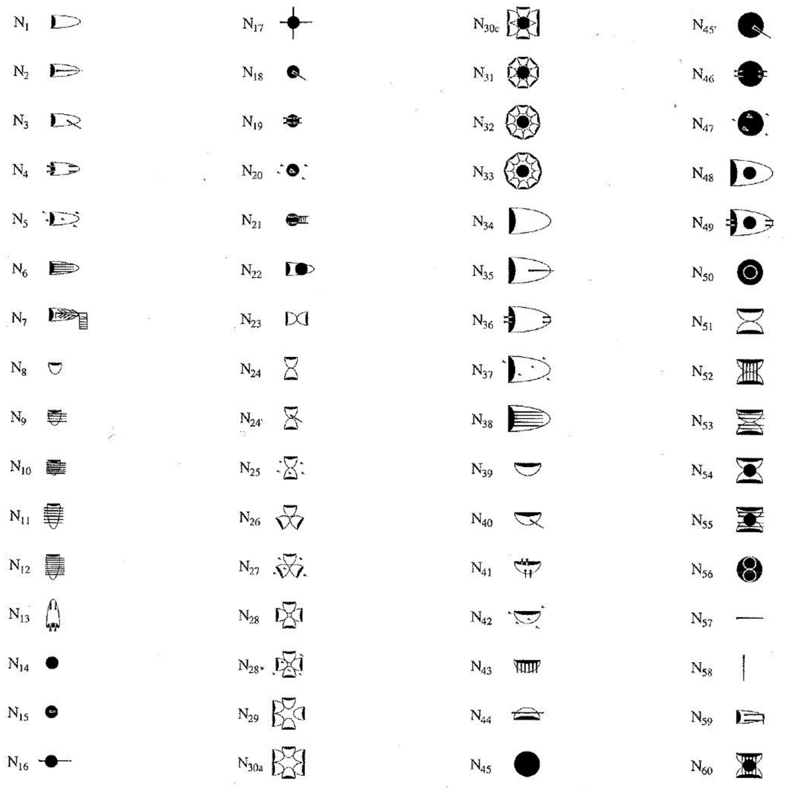
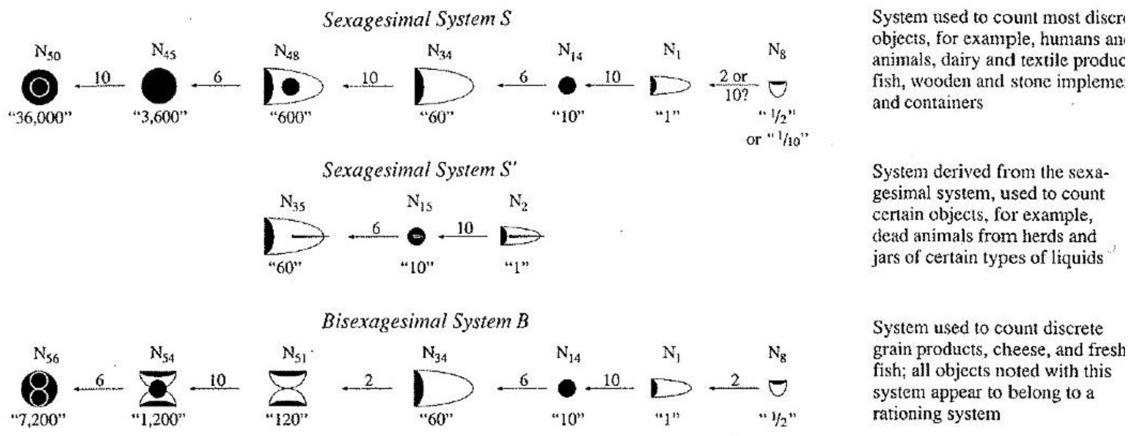


Figure 27. The numerical signs of the proto-cuneiform texts from Uruk.

26

Nissen et al. 1993, 26

One has to stress that establishing this list was only possible thanks to modern computer techniques, which enabled to systematically evaluate the clay tablets preserved in various sites and museums of the world. But, even more importantly, these computer techniques were decisive for unravelling the meaning of these signs. In the traditional literature, there had been contradictory attributions and unresolved questions, since with their means it had not been possible to evaluate systematically all these signs in the context of their use on the clay tablets. It is due in particular to the group of the researchers Peter Damerow, Robert Englund, and Hans Nissen to have undertaken this careful extensive investigation. Their research revealed that it was not just one number system, say the sexagesimal system, which one might expect, but that there were practiced several metrological systems and each system was specific for a certain class of objects. A further result was that numerous signs were not used in just one of these diverse metrological systems, but with differing meanings in several of them. This result resolved the inconsistencies in earlier attributions of meanings to the Uruk signs. Yet, it implied that the signs represented about 6.000 different meanings.
I will show here as example three different metrological systems, which show their specificity for object classes:



Nissen et al. 1993, 28

Thus, the first system counts mainly discrete objects, as humans and animals, but also fish and containers; the second counts dead animals and jars of some liquids. The third counts other discrete objects: some grains, but also fresh fish. Even more specific are the sign systems for various grains, in particular for barley, malt, oats, grouts. One remarks the importance of brewing beer.
In these early sign systems, the signs stand for classes of objects – thus for qualities – and for their quantities. They do not signify numbers, hence, but magnitudes. Researchers on the history of writing – a well known specialist is Denise Schmandt-Besserat (see Schmandt-Besserat 1996) – and researchers on the history of mathematics agree that number and scripture originated together, in the same socio-cultural setting. Eleanor Robson, researcher on Mesopotamian mathematics, formulated the consensus of both sides recently:

> "The temple administrators of Uruk adapted token accounting to their increasingly complex needs by developing the means to record not only quantities but the objects of account as well. Thus numeracy became literate for the first time in world history" (Robson 2008, 28).

One remarks a steady process of standardification in these metrological systems, which hence imply at the same time a greater abstraction of operations with quantities. A highly telling example of this process is given by a clay tablet where various amounts of different grains are added, thus making abstraction of their specific type of grain.

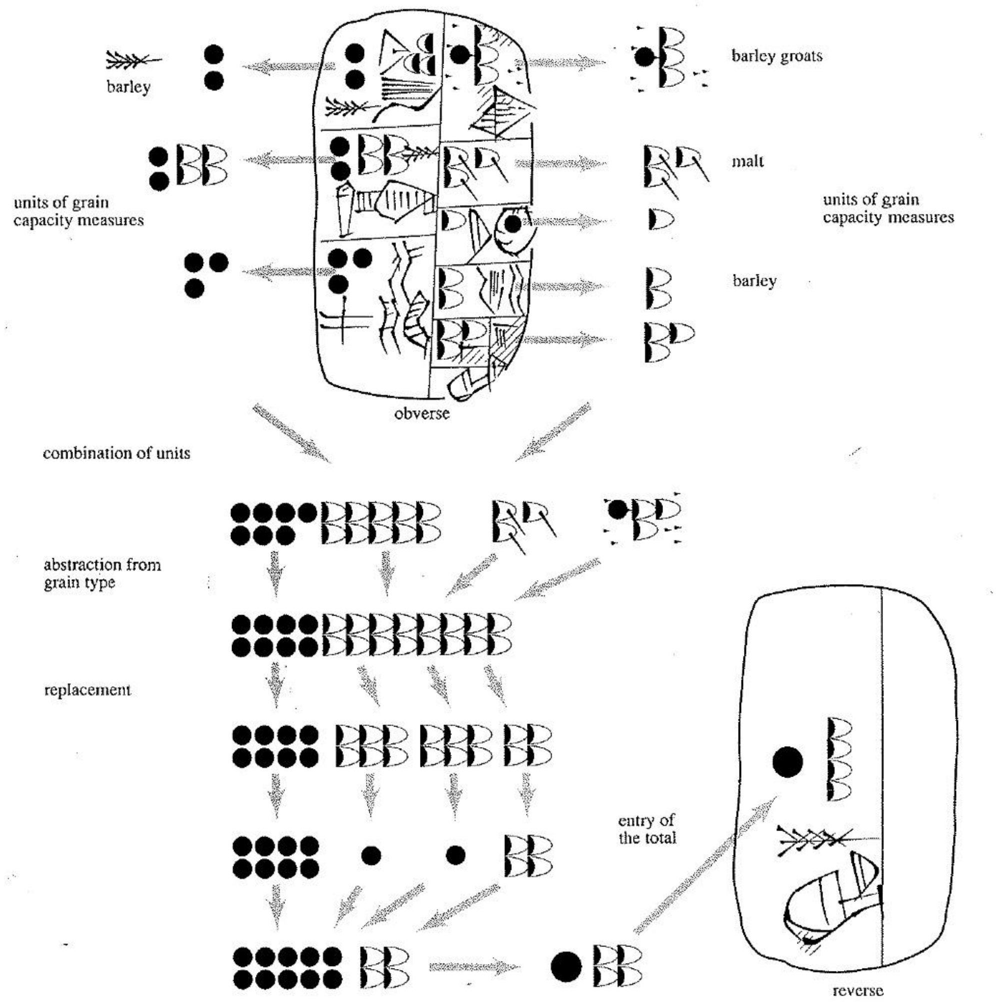


Figure 116. Complex proto-arithmetical summation involving replacement rules for symbols representing concrete units.

133

At first, the given units were indicated with the signs from the different systems for barley, barley grouts, and malt: It followed an operation by which the signs for units of barley grouts and of malt were transformed into the corresponding signs for barley. Having now homogeneous terms in just one metrological system, the various units were simplified, transforming the items into the respective higher units, thus ending with the sum of all products in a simple expression, in terms of just one quality of grain.

In the long run, the process of standardization of metrological systems and of abstraction from the qualities, with which one operated, continued in such a universal manner, that at the end of the process in the Old Babylonian civilization, it remained just two signs to indicate: no longer quantities, but now numbers – namely the signs for 1 and for 60 (and its higher powers):

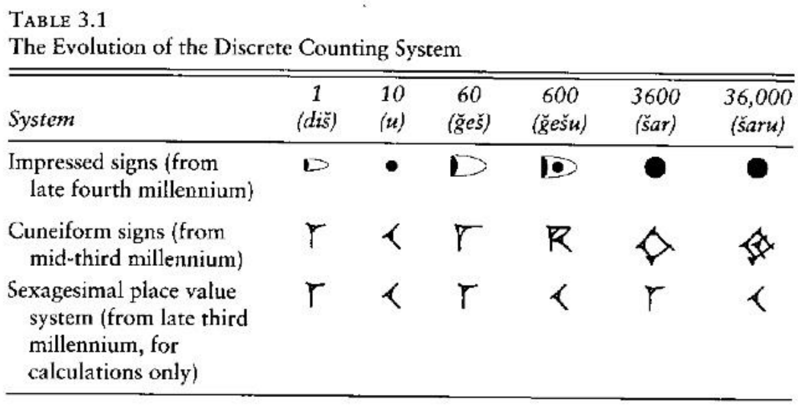
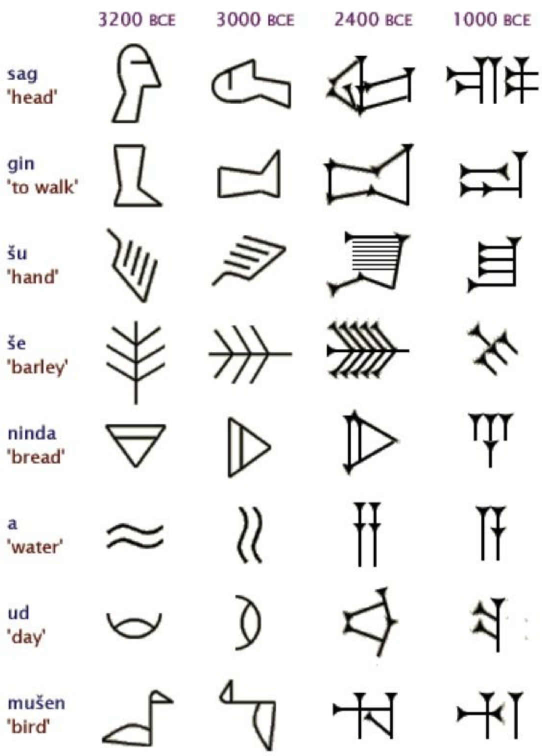


TABLE 3.1
The Evolution of the Discrete Counting System

System	1 (diš)	10 (u)	60 (ğeš)	600 (ğešu)	3600 (šar)	36,000 (šaru)
Impressed signs (from late fourth millennium)						
Cuneiform signs (from mid-third millennium)						
Sexagesimal place value system (from late third millennium, for calculations only)						

As is well known, this well developed number system suffered but one defect, the lack of a sign for zero, so that the place values were not unequivocally determinable. There are examples of clay tablets where Babylonian scribes had committed calculating errors, because they had not considered the empty space necessary for representing an empty unit.

Parallel to the process of transformation of magnitudes into numbers occurred the process of evolution of cuneiform signs fo writing words, from the multitude of early icons to composite forms of the simple wedge element:



The way to effective positional systems

The Sumerian-Babylonian scripture meant hence no alphabetic system; the apparently first alphabetic system was the Phoenician, emerging about 1.000 BC. The Phoenician alphabet became the model for the Greek alphabet, and the Greeks used it for a considerable improvement regarding the number system. The so-called number system of Milet applies the letters of the Greek alphabet as signs for numbers, too. Its improvement consists in that it suffers no ambiguity with regard to the values of the numbers; although not having a sign for zero, numbers can be written without any doubt about their positional value.

α	β	γ	δ	ε	ϛ	ζ	η	θ	
1	2	3	4	5	6	7	8	9	
ι	κ	λ	μ	ν	ξ	ο	π	ϟ	
10	20	30	40	50	60	70	80	90	
ρ	σ	τ	υ	φ	χ	ψ	ω	ϡ	
100	200	300	400	500	600	700	800	900	
͵α	͵β	͵γ	͵δ	͵ε	͵ϛ	͵ζ	͵η	͵θ	
1000	2000	3000	4000	5000	6000	7000	8000	9000	

The writing of the numbers occurs by juxtaposing the signs. Thus, for instance 867 is given as ωξζ and 807 as ωζ. This system needed 27 signs, but the Greek alphabet had only 24; to remedy this, one introduced three additional signs, from elder scripture variants and from Phoenician: the signs for 6, 90, and 900.

While the Egyptian system of numeration and the Roman, which followed its structure, did not present innovations in the direction of a more general positional system, it were the Indians who developed for the first time a decimal positional system with a sign for zero, by ca. 500 CE. Yet, the writing for this system, which became transmitted by the 9th century to the Islamic civilization and is therefore called the indo-Arabic number system, varied enormously and was for a long time not standardized. In particular, there was the West-Arab and the East-Arab variant; it was the West-Arab variant (no. 8 in the following table), which became received in Europe; the East-Arab one is no. 5:

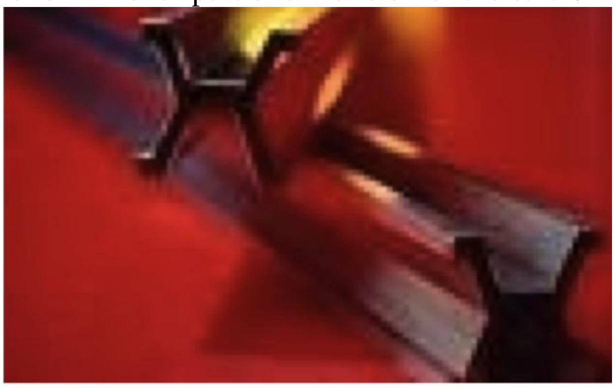
One has often related that the indo-Arab numbers were not accepted immediately in (Christian) Europe during the Middle Ages. In numerous historical accounts, the Catholic Church is accredited to have forbidden, in the 14th century, the use of these numbers. As a careful recent study has shown, the Church was not involved at all; there are no fundamental or ideological reasons. Instead, it had been the guild of commerciants in Florence, which had warned in 1299 against the use: due to the missing standardization of writing them, there was an evident danger of frauds (Lüneburg 2008, 106-109).
Actually, hesitation or resistance was not restricted to Western Europe. Although the name ‘Arab numbers’ or ‘indo-Arab numbers’ suggests that they became the dominantly applied number system in the Islamic civilization, this is not at all the case. In reality, this new system was not generally accepted; two traditional systems remained dominant, and that until the 19th century. It was, on the one hand, the sexagesimal system and, on the other hand, an alphabetical system, which was a transposition of the Greek system of Milet.

ا	ب	ج	د	ه	و	ز	ح	ط	
a	b	g	d	e	w	z	ḥ	ṭ	
1	2	3	4	5	6	7	8	9	
ي	ك	ل	م	ن	س	ع	ف	ص	
y	k	l	m	n	s	ʿī	f	ṣ	
10	20	30	40	50	60	70	80	90	
ق	ر	ش	ت	ث	خ	ذ	ض	ظ	غ
q	r	sh	t	th	kh	dh	ḍ	ẓ	gh
100	200	300	400	500	600	700	800	900	1000

Introducing the metric system

Although the indo-Arab numbers were effectively accepted and used in the West-European cultures since the introduction of the printing press – all arithmetic and mathematics books printed used the “new” numbers -, a decisive advantage of this system was not understood and applied: to be a decimal system. The units for weights and measures had no decimal structure for the sub-units, but highly complicated systems for their sub-units. Think of the old subdivisions of a pound – one pound being 20 shilling and one shilling being 12 pence – or the subdivisions of yard into feet and inches, or of ounces, etc. Moreover, there was no uniformity for the absolute values of the measures - not only not for the territory of a given country, but also not for entire regions; rather, the values might be different from one town to the next one. Evidently, these incompatible weights and measures constituted major obstacles for all kinds of commerce.
It became one of the major projects of the French Revolution to standardize and universalize weights and measures. The Republic organized even the first international congress on science, in 1798 and 1799 in Paris, to establish the exact definitions of the

new measures, in particular of the meter (Crosland 1969). A preliminary definition of the new units of length had already been established and published in 1793. The conception for the new measures was to have them based on “natural” data; thus, the meter was defined as one-millionth part of a meridian of the earth.



The meter prototype, in its improved version of 1889

Extensive and sophisticated astronomical observations were realized to achieve exact values. The 1793 publication gave also the concordance with the old length units for the town of Paris:

TABLEAU DU NOUVEAU SYSTÈME DES POIDS ET MESURES
ET DE LEURS DÉNOMINATIONS
(Annexé au décret de la Convention nationale du 1er août 1793, an II de la République.)

Mesures linéaires.

Unité prise dans la nature					
			Valeurs en toises et pieds de Paris.		
	10 000 000	Quart du méridien			5 132 430 toises
	1 000 000				513 243
	100 000	*Grade* ou degré décimal du méridien.			51 324
	10 000				5 132
	1 000	*Millaire*			513
	100		307 pieds	11 pouces	4 lignes.
	10		30	9	6,4
Unité linéaire. Dix-millionième partie du quart du méridien	1 MÈTRE		3	0	11,44
	1/10 Décimètre		0	3	8,344
	1/100 Centimètre		0	0	4,434
	1/1000 Millimètre		0	0	0,443

NOTA. - Les besoins de la société n'exigeant point nécessairement des noms particuliers pour tous les multiples décimaux du mètre, on s'est abstenu de leur en donner. Ainsi, au-dessus du mètre, on compte, sans aucune nouvelle dénomination, jusqu'à mille mètres, que l'on prend, sous le nom de miliaire, pour l'unité des grandes distances ou des mesures itinéraires.

In 1793, the measure for capacity was not yet the liter and for weights not yet the kilogram, but it were “pinte” and “grave” (referring to “heavy”). The intention of the reformers for universalizing a completely metric decimal system were so radical that also the week with seven days was changed to the *décade*, with ten days, and the circle became divided in 400 degrees – so that a right angle had 100 degrees. While the decade did not survive an extended period, the 400 degrees were practiced for quite a period.

Telling is, however, the destiny of the metric system. As is well known the French Revolution was effected by the “tiers état”, by the bourgeoisie, against the feudal powers and the cleric. The famous three key words for the Revolution – *liberté, égalité, fraternité* – were originally a bit different: *liberté*, *égalité*, *propriété*. They expressed thus a clear socio-economic program: not only individual liberty, but assuring also free economic exchanges, without impediments for property - hence, there should be freedom for commerce, too: without any obstacles. For free circulation of goods, obstacles like the plethora of local measures, should be substituted by a universal standardized system of weights and measures. One of the objectives of the international Science Congress 1798/99 in Paris was therefore to disseminate the metric system to all other countries. Due to the conflict of France with Great Britain, the British neither participated nor accepted the metric system. Acceptance was achieved for countries in some way allied with France, and even for those its practice was in general discontinued after the fall of the French Empire.

Despite the strong political intention to generally practice the metric system, the extraordinary historical phenomenon revealed that it met strong resistance by those who should mainly apply it, in particular the merchants. Century long traditions proved to have internalized the old systems, although impractical and uncomfortable, in such a manner that simple laws prescribing the application of the metric system were not effective. A law of 4 November 1800 tried to enhance the acceptance of the metric system by allowing the substitute the new names by the old ones – thus pound instead of kilogram, pinte instead of liter, and finger instead of centimetre. In 1812, it was even allowed to return to the old subdivisions. For lengths, for instance, it was allowed to use:

> “une toise de 2 mètres, se divisant en 6 pieds; le pied valant ainsi un tiers du mètre, se divisant en 12 pouces, le pouce en 12 lignes”.

It resulted an enormous confusion and considerable frauds. Nevertheless, it was only by a law of 4 July 1837, that the strict application of the metric system was definitely prescribed for France, from 1840 on.

In other European countries, where governments in general had not been really active to introduce the metric system, popular resistance endured for extended periods. Germany with its multitude of independent countries throughout the 19th century presents revealing cases. A particularly telling one concerns the duchy of Holstein, north of Hamburg. After a law of currency reform in 1854, an arithmetic schoolbook showed the somewhat reformed measures. On the one hand, despite the small regions there, they were even specific for its sub-regions. For instance, for grains, there are the measures for Hamburg and for Dithmarschen. For Hamburg the measures are different for wheat, rye, and peas on the one hand, and for barley and cats on the other hand. The first group is measured in Last, and this subdivides in 3L. For the second group, one *Last* divides into 2 *Wispel*. For both groups, one *Wispel* divides into 10 *Scheffel* (bushel); but then the *Scheffel* divides again differently: into 2 *Fass* for the first group and into 3 *Fass* for the second. Then, one *Fass* divides again jointly: into 2 *Himpten*, and one *Himpten* into 4 *Spint*. For Dithmarschen, measuring linseed is done by dividing one *Tonne* (ton) into 6 *Scheffel* and this into 10 and two thirds *Kannen* (Kroymann 1854, 340).

Regarding measures of areas, the following extract from that schoolbook evidences their enormous differentiation according to even much smaller regions: for the towns of Hamburg and Glückstadt, for the *Ämter* Steinburg and Wilster Marsch and for

Dithmarschen even different for its south and its north. The principal area unit is called *Morgen* (acre).

Verschiedenes Landmaß.

Das Hamburgische Landmaß.

1 Morgen ist eine Fläche von 120 Ruthen lang und 5 Ruthen breit, und hält also 600 □Ruthen.
1 Morgen hält 30 Scheffel oder Saat, à 20 □Ruthen.

Das Glückstädtische Landmaß nach dem Hamburger oder Schlesw. u. Holst. Längenmaß.

1 Morgen ist eine Fläche, welche 120 Ruthen lang und 3 Ruthen breit ist, und hält also 360 □Ruthen.

Das Landmaß im Amte Steinburg oder der Wilster Marsch nach dem Schlesw. u. Holst. Längenmaß.

1 Morgen ist eine Fläche, die 120 Ruthen lang und 3 Ruthen 12 Fuß breit ist, und also 450 □Ruthen hält.

Das Landmaß in Süderdithmarschen nach dem dasigen Längenmaß.

1 Morgen hält 15 Scheffelland, à 40 □Ruthen.
1 Ruthe hält 16 Fuß, à 16 Finger Längenmaß.
25 Süderdithmarscher Ruthen sind gleich 26 Hamburger Ruthen Längenmaß, und also ist ein Süderdithmarscher Morgen oder 600 □Ruthen (ziemlich genau) = 649 Hamb. □R.

Das Landmaß in Norderdithmarschen ist ein kleines und ein großes Maß, und zwar das kleine im Heider oder südlichen District, und das große im Lundener oder nördlichen District.

Im **südlichen** District hält 1 Morgen Marschland 20 Scheffelsaat, à 30 □Ruthen, oder 600 □Ruthen.
1 Ruthe hält 16 Fuß, à 16 Finger Längenmaß.
Im **nördlichen** District hält 1 Morgen Marschland 16 Scheffelsaat, à 30 □Ruthen, oder 480 □Ruthen.
1 Ruthe hält 18 Fuß, à 18 Finger Längenmaß.
Das Fußmaß des kleinen und großen Maßes ist sich gleich.

Kroymann 1854, 341

The digital era

While scripture and numbers had originated jointly, their further development had occurred basically in a different manner, although the Greek and the Arab numeration system showed affinities again. Both strands converged again, however, with the rise of the digital area.

Having established the theoretical foundations for digitalizing entire mathematics can be attributed to David Hilbert. In his famous Zürich conference of 1917 on axiomatic thinking, he reported about the progress of his program of arithmetizing mathematics. Satisfied, he asserted that the entire extended theory of Euclidean geometry can be constructed by means of analysis. And the analysis can be shown as based on the theory of real numbers. This theory, in turn, can be shown to be free of contradictions when the theory of entire numbers is free of contradictions. The only open problem is, as he pointed out, to show that the axioms of the entire umbers are free of contradictions, which should be the task of logic (Hilbert 1918, 406 ff.). Although this last step did not succeed, dynamic software demonstrates vigorously how mathematics can be digitalized.

Regarding scripture, it proved to be even more easy to digitalize words, to transform them into chains of the two signs 0 and 1: one had to dissect words into their letters, find for instance the ascii code of these letters and then establish their binary codes. By the evolution of these procedures, words and numbers documented to be of the same nature: certain binary codes.

As in all our historical cases so far, also here we remark some resistance against a complete universalization. Digitalization of words supposes that words are constituted by means of an alphabet. Yet, there are languages not based on an alphabet – and these are not minor languages, but Chinese and Japanese figure among them, being based on icons. And there are several thousands of icons! On the other hand, Chinese and Japanese people use computers for composing texts. Actually, there are no ways for coding the icons directly; the means developed so far consist of software programs, which handle the icons as graphical elements. One of these software approaches for Chinese language is the Cangjie input method; it corresponds to a simplified use of Chinese.

Coding of "倉頡輸入法" (i.e. Cangjie method) in traditional Chinese characters

Chinese icons are understood as composed of "radicals", there are 24 such radicals; the method is based on a geometric decomposition of the icons. The decomposition becomes specialized by 76 auxiliary shapes: often rotated or transposed versions of the radicals. Radicals and auxiliary shapes can be coded thus that one can enter them by operating with the standard keyboard. The use of these softwares is not easy, so far. For Cangjie, for instance, one has to know the names of all radicals and also of all their auxiliary shapes. Moreover, one has to be familiar with the decomposition rules for the icons.

There are still steps to be mastered!

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